Exercise 31

In Exercises 29 to 31, use vector methods to describe the given configurations.

The plane determined by the three points (x_0, y_0, z_0), (x_1, y_1, z_1), and (x_2, y_2, z_2)

Solution

Only two linearly independent vectors are needed to get a plane. Subtract the position vectors from one another to obtain them.

$$\mathbf{r}_1 = (x_1, y_1, z_1) - (x_0, y_0, z_0) = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$
$$\mathbf{r}_2 = (x_2, y_2, z_2) - (x_0, y_0, z_0) = (x_2 - x_0, y_2 - y_0, z_2 - z_0)$$

The linear combination of $\mathbf{r}_1$ and $\mathbf{r}_2$ results in an entire plane that passes through the origin ($s = 0$ and $t = 0$).

$$\begin{aligned}\mathbf{r}(s,t) &= s\mathbf{r}_1 + t\mathbf{r}_2 \\ &= s(x_1 - x_0, y_1 - y_0, z_1 - z_0) + t(x_2 - x_0, y_2 - y_0, z_2 - z_0)\end{aligned}$$

By adding (x_0, y_0, z_0) to $\mathbf{r}(s,t)$, this plane can be made to pass through (x_0, y_0, z_0) when $s = 0$ and $t = 0$ instead.

$$\{(x_0, y_0, z_0) + s(x_1 - x_0, y_1 - y_0, z_1 - z_0) + t(x_2 - x_0, y_2 - y_0, z_2 - z_0),\ s \in \mathbb{R},\ t \in \mathbb{R}\}$$